

## 17.5.2021: Decomposition of the intersection multiplicity into this $\Omega$ sequence is different from blowup

When finding intersection multiplicity via blowup, we end up with sequence of numbers. Each step of the blowup algorithm gives us one member of this sequence and their sum is the intersection multiplicity.

Identical blowup sequence does not imply identical  $\Omega$  sequence. Identical  $\Omega$  sequence does not imply identical blowup sequence. This means these two algorithms are in some sense different, but I'm not sure whether it is a good thing or a bad thing.

### EXAMPLE 1: intersections with different blowup sequences, but identical $\Omega$ sequences

Let

$$F_1 = x^5 - y^8 \quad (1)$$

$$G_1 = x^3 - y^4 \quad (2)$$

- **BLOWUP:**  $I_O(F_1, G_1) = 5 \cdot 3 + 3 \cdot 1 + 2 \cdot 1 = 15 + 3 + 2 = 20$
- **$\Omega$ :**  $I_O(F_1, G_1) = 1 + 2 + 3 + 3 + 3 + 3 + 2 + 2 + 1 = 20$

$$F_2 = x^5 - y^{11} \quad (3)$$

$$G_2 = x^3 - y^4 \quad (4)$$

- **BLOWUP:**  $I_O(F_2, G_2) = 5 \cdot 3 + 5 \cdot 1 = 15 + 5 = 20$
- **$\Omega$ :**  $I_O(F_2, G_2) = 1 + 2 + 3 + 3 + 3 + 3 + 2 + 2 + 1 = 20$

### EXAMPLE 2: intersections with identical blowup sequences, but different $\Omega$ sequences

Let

$$F_1 = x^2 - y^5 \quad (5)$$

$$G_1 = x^4 - y^7 \quad (6)$$

- **BLOWUP:**  $I_O(F_1, G_1) = 2 \cdot 4 + 2 \cdot 3 = 8 + 6 = 14$
- **$\Omega$ :**  $I_O(F_1, G_1) = 1 + 2 + 2 + 2 + 2 + 2 + 2 + 1 = 14$

$$F_2 = x^2 - y^5 \quad (7)$$

$$G_2 = x^4 + xy^5 - y^7 \quad (8)$$

- **BLOWUP:**  $I_O(F_2, G_2) = 2 \cdot 4 + 2 \cdot 3 = 8 + 6 = 14$
- **$\Omega$ :**  $I_O(F_2, G_2) = 1 + 2 + 2 + 2 + 2 + 2 + 1 + 1 + 1 = 14$