

### 13.8.2021: Upper bound of intersection multiplicity

Since the pyramids idea didn't work out, I've put my hopes into my dear friend  $\mathcal{O}$ . Let  $z_0$  be the step at which the algorithm ends. It is the first position where the  $\mathcal{O}$  sequence reaches 0.

**Example 1.** *Let*

$$F = x^2 - y^5 \tag{1}$$

$$G = x^4 + xy^5 - y^7 \tag{2}$$

Then the corresponding sequence is  $\mathcal{O} = [1, 2, 2, 2, 2, 2, 1, 1, 1, 0, 0, 0, \dots]$ . (In the version below, 1 represent interesting 1s and  $\cdot$  represent boring 1s.)

1	2	2	2	2	2	1	1	1	0	0	0	...
=	=	=	=	=	=	=	=	=	=	=	=	...
1	2	2	2	1	0	-1	-2	-3	-4	-5	-6	...
			1	1	·	·	·	·	·	·	·	...
				1	1	·	·	·	·	·	·	...
							1	·	·	·	·	...
								1	·	·	·	...
										·	·	...
											·	...

In this case,  $z_0 = 10$ ,

It is be also defined as the smallest  $z$ , such that  $\mathcal{O}/(F, G, I^z) \cong \mathcal{O}/(F, G)$ . The intersection multiplicity have the following property:

$$I_O(F, G) \leq m \cdot n + t \cdot (z_0 - n - m), \tag{3}$$

(here  $m$  and  $n$  are the multiplicities of  $F$  and  $G$  at  $O$ , and  $t$  is the number of their common tangents at  $O$ ) This is a direct consequence of how many "ones" can we fit into the sequence before it ends.

If we could find some pretty upper bound for  $z_0$ , it would give us an upper bound for the intersection multiplicity itself. The examples suggest that the bound for  $z_0$  it could be possibly a fairly low number, something like  $(\deg(F) + \deg(G))$ , or even  $(m + \max\{\deg(F), \deg(G)\})$  or  $(t + \max\{\deg(F), \deg(G)\})$ .