3.1.2022: Maybe making these into exact sequences (starting with 0) can be a good idea (an example)

Let F and G be defined by

$$\begin{aligned} F &= x^3 - y^5, \\ G &= x^5 - y^8. \end{aligned} \tag{1}$$

link na graf: `https://www.desmos.com/calculator/lvywu1yc6n`

Therefore $m = 3$ and $n = 5$. Then $\ker\psi_0 = \cdots = \ker\psi_5 = (0,0)$ and

$$\begin{aligned}
\ker\psi_6 &= D_0(x^2,1) \\
\ker\psi_7 &= D_1(x^2,1) + D_0(x^2,1) \\
\ker\psi_8 &= D_2(x^2,1) + D_1(x^2,1) \\
\ker\psi_9 &= D_3(x^2,1) + D_2(x^2,1) + D_0(x^3+y^5,x) \\
\ker\psi_{10} &= D_4(x^2,1) + D_3(x^2,1) + D_1(x^3+y^5,x) \\
\ker\psi_{11} &= D_5(x^2,1) + D_4(x^2,1) + D_2(x^3+y^5,x) + D_0(x^4+x^2y^3+xy^5, x^2+y^3) \\
\ker\psi_{12} &= D_6(x^2,1) + D_5(x^2,1) + D_3(x^3+y^5,x) + D_1(x^4+x^2y^3+xy^5, x^2+y^3) \\
\ker\psi_{13} &= D_7(x^2,1) + D_6(x^2,1) + D_4(x^3+y^5,x) + D_2(x^4+x^2y^3+xy^5, x^2+y^3) + D_0(x^5-y^8, x^3-y^5) \\
\ker\psi_{14} &= D_8(x^2,1) + D_7(x^2,1) + D_5(x^3+y^5,x) + D_3(x^4+x^2y^3+xy^5, x^2+y^3) + D_1(x^5-y^8, x^3-y^5) + D_0(x^5-y^8, x^3-y^5) \\
\ker\psi_{15} &= \cdots
\end{aligned} \tag{2}$$

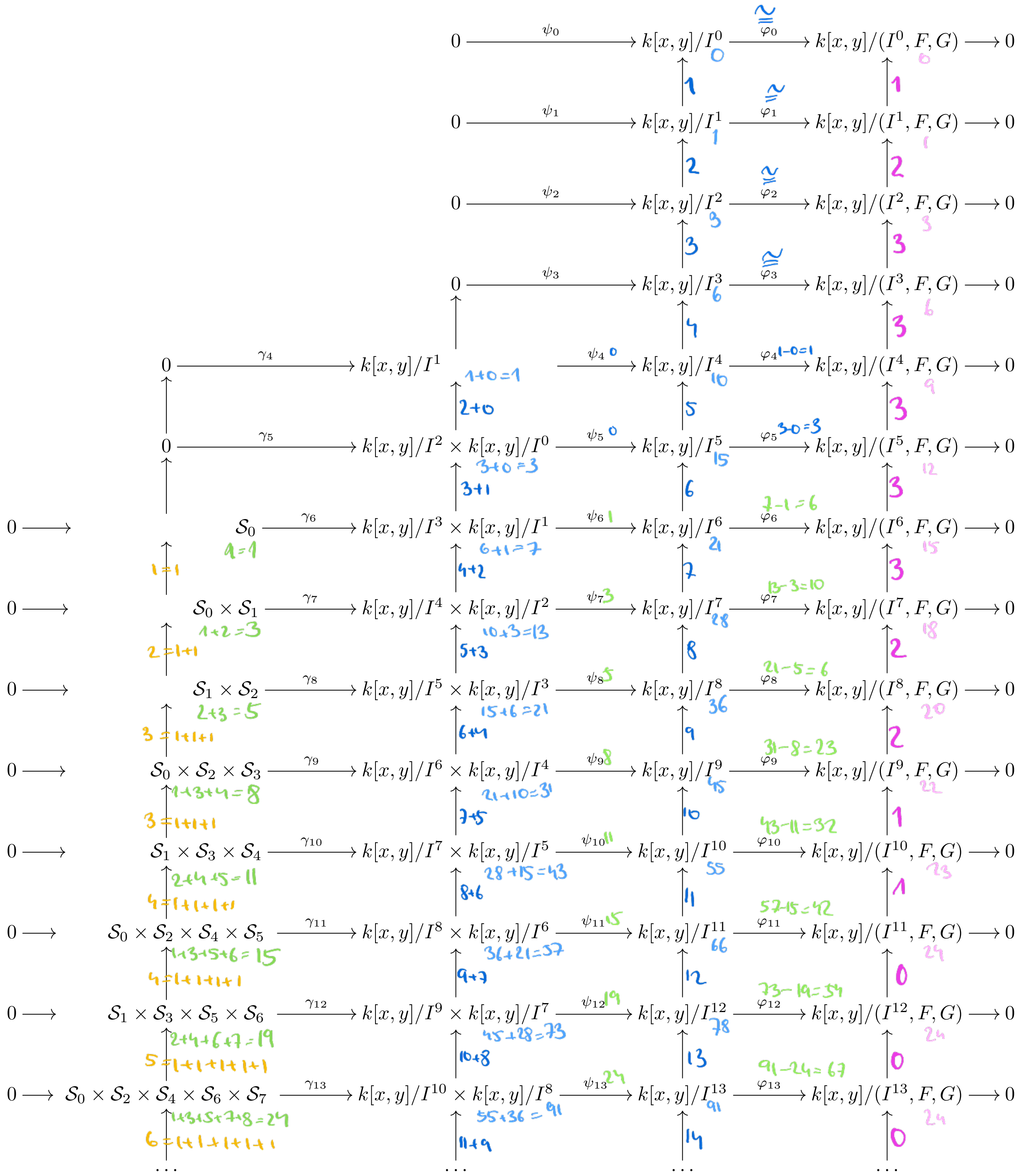
The ψ table is

$$\begin{array}{ccccccccccccccc}
1 & 2 & 3 & 3 & 3 & 3 & 3 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & \cdots \\
= & = & = & = & = & = & = & = & = & = & = & = & = & = & \cdots, \\
1 & 2 & 3 & 3 & 3 & 2 & 1 & 0 & -1 & -2 & -3 & -4 & -5 & -6 & \cdots, \\
 & & & & & 1 & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots, \\
 & & & & & & 1 & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots, \\
 & & & & & & & & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdots, \\
 & & & & & & & & & & 1 & \cdot & \cdot & \cdot & \cdots, \\
 & & & & & & & & & & & & \cdot & \cdot & \cdots, \\
 & & & & & & & & & & & & & \cdot & \cdots,
\end{array} \tag{3}$$

Therefore

$$I_O(F,G) = mn + \sum a_i = 3 \cdot 5 + 3 + 3 + 2 + 1 = 24. \tag{4}$$

So we can construct this diagram where each row is an exact sequence. Here, I is the ideal (x,y), and $\mathcal{S}_i = I^i/I^{i+1}$. The maps φ_i (and also all the vertical maps) are canonical, and $\psi_i(A,B) = FA - BG$. The maps γ_i are defined below.



The γ maps are obviously based on kernels of ψ maps.

a sequence

depends only on m and n

units + dots

$$
\begin{aligned}
\gamma_6\colon && D_0 &\longrightarrow D_0(x^2,1) && = (D_0x^2, D_0)\\
\gamma_7\colon && (D_0,D_1) &\longrightarrow D_1(x^2,1)+D_0(x^2,1) && = (D_1x^2+D_0x^2, D_1+D_0)\\
\gamma_8\colon && (D_1,D_2) &\longrightarrow D_2(x^2,1)+D_1(x^2,1) && = \cdots\\
\gamma_9\colon && (D_0,D_2,D_3) &\longrightarrow D_3(x^2,1)+D_2(x^2,1)+D_0(x^3+y^5,x)\\
\gamma_{10}\colon && (D_1,D_3,D_4) &\longrightarrow D_4(x^2,1)+D_3(x^2,1)+D_1(x^3+y^5,x)\\
\gamma_{11}\colon && (D_0,D_2,D_4,D_5) &\longrightarrow D_5(x^2,1)+D_4(x^2,1)+D_2(x^3+y^5,x)+D_0(x^4+x^2y^3+xy^5,x^2+y^3)\\
\gamma_{12}\colon && (D_1,D_3,D_5,D_6) &\longrightarrow D_6(x^2,1)+D_5(x^2,1)+D_3(x^3+y^5,x)+D_1(x^4+x^2y^3+xy^5,x^2+y^3)\\
\gamma_{13}\colon && (D_0,D_2,D_4,D_6,D_7) &\longrightarrow D_7(x^2,1)+D_6(x^2,1)+D_4(x^3+y^5,x)+D_2(x^4+x^2y^3+xy^5,x^2+y^3)+\\
&&&\quad + D_0(x^5-y^8,x^3-y^5)\\
\gamma_{14}\colon && (D_0,D_1,D_3,D_5,D_7,D_8) &\longrightarrow D_8(x^2,1)+D_7(x^2,1)+D_5(x^3+y^5,x)+D_3(x^4+x^2y^3+xy^5,x^2+y^3)+\\
&&&\quad + D_1(x^5-y^8,x^3-y^5)+D_0(x^5-y^8,x^3-y^5)\\
\gamma_{15}\colon && &\cdots
\end{aligned} \tag{5}
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