14.7.2023 special case of a special case

So far we know that if some tangent T is t times in ${}_0L$ and does not occur in ${}_1L$, then the contribution of the tangent T into the intersection multiplicity is t.
We would like to extend this to ${}_1L$. Therefore we would like to prove that if some T is t_0 times in ${}_0L$ and t_1 times in ${}_1L$ and does not occur in ${}_2L$, then the contribution of the tangent T into the intersection multiplicity is $t_0 + t_1$.

special case

Let $F = F_m + F_{m+1} + \cdots$ and $G = G_n + G_{n+1} + \cdots$ be a pair of curves intersecting at the origin, such that

$$y \mid F_m = {}_0L = {}_1L,$$
$$y \nmid {}_2L$$

Let y be a tangent of both F and G of multiplicity s. Then F and G are of the form

$$\begin{aligned} F &= \left(y^s L_v L_{m-v-s}\right) + \left(F_{m+1}\right) + \left(F_{m+2}\right) + \cdots \\ G &= \left(y^s L_v L_{m-v-s} Z_{n-m}\right) + \left(Z_{n-m} F_{m+1} + Z_{n-m+1} y^s L_v L_{m-v-s}\right) + \\ &\quad + \left(Z_{n-v+2} L_v + Z_{n-m+1} F_{m+1} - Z_{n-m} F_{m+2}\right) + \cdots \end{aligned}$$

for any numbers v, s (such that $v + s \le m$) and polynomials L_v, L_{m-v-s}, F_{m+1}, F_{m+2}, Z_{n-m}, Z_{n-m+1}, Z_{n-v+2} such that

- $y \nmid L_v$
- $y \nmid L_{m-v-s}$
- $y \nmid Z_{n-2-v}$
- $\gcd(Z_{n-2-v}, L_{m-v-s}) = 1$.

Then $\mathcal{L} = (y^s L_v L_{m-v-s}, \quad y^s L_v L_{m-v-s}, \quad L_v, \cdots)$.

Proof (by direct calculation) in PAP: 23_07_14_A