21.7.2023 Branch of a curve F with the tangent y of the multiplicity 1

Let y a tangent of the curve $F = F_m + F_{m+1} + \cdots$ of multiplicity 1. Without loss of generality, let the coefficient of $x^{m-1}y$ be 1. We denote $F_{m+j} = \sum f_i^0 x^{m+j-i} y^i$, therefore

$$\begin{aligned} F &= F_m + F_{m+1} + F_{m+2} + \cdots = \\ &= 0 + f_1^0 x^{m-1} y + f_2^0 x^{m-2} y^2 + \cdots + f_m^0 y^m + \\ &\quad + f_0^1 x^{m+1} + f_1^1 x^m y + \cdots + f_{m+1}^1 y^{m+1} + \\ &\quad + f_0^2 x^{m+2} + f_1^2 x^{m+1} y + \cdots + f_{m+2}^2 y^{m+2} + \\ &\quad + \cdots , \end{aligned}$$

Then there is only one branch of F with the tangent y and its parametrization is of the form

$$b(t) = \left(t, -f_0^h t^{h+1} - V(H) t^{h+H+1}\right) = \left(t, t^{h+1}\left(-f_0^h - V(H) t^H\right)\right) = \tag{1}$$

where h is the smallest number such that f_0^h is nonzero. $V(H) = V(e,i)$ (for $H = eh + i$, $e, i \geq 0$, $i < h$) is defined as

$$V(H) = V(e,i) = \sum_{j=0}^{e+1} \left(-f_0^h\right)^{e+1-j} f_{e+1-j}^{jh+i}$$

(if $(jh+i) - m > e + 1 - j$ then $f_{e+1-j}^{jh+i} := 0$).
The number H is the smallest integer such that $V(H)$ is nonzero.

PROOF: by direct computation in 23_07_25_ A